EDITOR'S ENDNOTES

The following was submitted to us by Rolfdieter Frank and Harald Riede.

We would like to add an important reference for our article "Hyperplane Sections of the n-Dimensional Cube" (Vol. 119, No.10, December 2012): J. L. Marichal and M. J. Mossinghoff, "Slices, slabs, and sections of the unit hypercube," *Online Journal of Analytical Combinatorics*, January 2008. The authors derive several volume formulas for sections of the hypercube. In particular, they obtain an elementary formula for arbitrary hyperplane sections, which is equivalent to the formula in our article. Their references include a number of papers where similar formulas have come up, dating to Pólya's thesis in 1912.

Thomas H. Foregger offers the following comment about the paper "A new proof of a classical formula," by Habib Bin Muzaffar (Vol. 120, No. 4, April 2013).

Another simple proof of this result can be found in

http://www.jstor.org/stable/pdfplus/2320072.pdf

which uses just a mean value theorem for integrals.

Gerald Folland offers the following observation about the filler piece "Differentiating Iteratively," by Philip Mummert and Ken Constantine (Vol. 120, No. 6, June–July 2013).

As a point of information: This theorem is an exercise in Section 2.3 (p. 70) of my book *Advanced Calculus* (Prentice Hall, 2002), and the proof is the one suggested in my hint. I agree, this observation deserves to be more widely known.

With respect to the same filler piece, Gerry Bilodeau offers the following.

Two papers closely related to this filler piece are "An exponential rule," *College Mathematics Journal* (September 1993), pp. 350–351 (with a similar narrative although a different proof) and in "Logarithmic differentiation: two wrongs make a right," *College Mathematics Journal* (November 2004), pp. 388–390. Both appeared in the *Calculus Collection*, Mathematical Association of America, 2010.

We received the following comments from Robin Chapman concerning the paper "A Probabilistic Proof of a Binomial Identity," by Jonathon Peterson (Vol. 120, No. 6, June–July 2013).

I very much enjoyed this article about applying conditional probability to prove this identity. The author observes that the Chu–Vandermonde identity and the Rice integral

http://dx.doi.org/10.4169/amer.math.monthly.120.10.954

could be used to prove the identity. But surely more natural and elementary methods to prove this are via partial fractions or via the calculus of finite differences. For instance, expanding

$$\frac{1}{x(x+1)(x+2)\dots(x+n)}$$

in partial fractions gives the identity immediately (with x in place of theta). Also, iterating the operation $f(x) \mapsto f(x) - f(x+1)$ n times gives $\sum_{k=0}^{n}(-1)^k\binom{n}{k}f(x+k)$. Applying this to $f(x) = 1/x$ and using induction also proved the identity. With a bit more effort, the generalizations in section 3 also yield to finite differences.

The following was submitted by Randy Schwartz regarding the paper "Origin and Evolution of the Secant Method in One Dimension," by Richard Tapia and Joanna Papakonstantinou, which appeared in the June–July 2013 issue.

The article states that when ancient Egyptians dealt with relationships of the form $ax + b = c$, "They gave instructions on how to obtain the solution using the relation

$$x = \frac{x_0 e_1 - x_1 e_0}{e_1 - e_0}.$$

This method for solving for x is now most commonly referred to as the *Rule of Double False Position*" (p. 508). The authors conclude, "…the Rule of Double False Position dates back to the 18th century BC…" (pp. 512–513). In fact, however, no instance of any double false position technique being used in ancient Egypt, with or without "instructions," is known to exist. The problems from Smeur cited on 508–509 are not from Egypt but from J. van der Schuere's arithmetic from Haarlem in 1611 AD.

The article also leaves readers with the impression that isāb al-khaā'ayn (the double false position method that was transmitted from the Arab world to Europe in medieval times) might have first reached the Middle East from China and/or India (pp. 509–511). While a double false position method was certainly used in Chinese antiquity, it was so different from the Arab method that this is not likely an instance of borrowing, as I showed in detail in a 2004 paper ("Issues in the origin and development of isāb al-Khaā'ayn (calculation by double false position)," *Actes du Huitième Colloque Maghrébin sur l'Histoire des Mathématiques Arabes, Tunis, les 18-19-20 Décembre 2004* (Tunis: Tunisian Association of Mathematical Sciences, 2006), pp. 275–296).

Professors Papakonstantinou and Tapia offer the following response.

We thank Professor Schwartz for his interest in our recent paper. Moreover, we view him as an expert in the history of the rule of double false position and in the general area of Arab and Islamic contributions to science. Our two statements brought into question by Professor Schwartz were made as considered opinions based on our readings. However, the situation could well be exactly as Professor Schwartz has described. The establishment of correct history is often far more challenging than is the establishment of correct mathematics.

José Hernández Santia offers the following observations about the paper "Variations on a theme: Rings satisfying $x^3 = x$ are commutative" by Stephen Buckley and Desmond MacHale, which appeared in the May 2013 issue.

In the introduction to the paper, its authors mentioned that "Herstein is said to have remarked that one exercise in his book gave rise to more correspondence from the readers than all other items put together ...." Then, they added that the exercise that originated such an amount of correspondence from the readers of the *Topics in Algebra* was exercise *19 (throughout the letter, we stick to the numbering of exercises in the second edition of the book [**2**]) in section 3.4 (Ideals and quotient rings): Let R be a ring in which $x^3 = x$ for every x in R. Prove that R is a commutative ring. Certainly, this "relationship" between the exercise to which the article was devoted and the aforementioned claim of the late Prof. Herstein may have caught the imagination of many a reader of the latest issue of the MONTHLY. Nevertheless, it has to be noted that it actually was exercise **26 of section 2.5 (A counting principle) that gave rise to the amount of correspondence which bedazzled Prof. Herstein. This exercise **26 asks for a proof of the following assertion: If a commutative group G has subgroups of order m and n, respectively, then it also has a subgroup whose order is the least common multiple of m and n. In [**2**, p.48], Prof. Herstein would insert the following comment just below the statement of the exercise: "Don't be discouraged if you don't get this problem with what you know up to this stage. I don't know anybody, including myself, who has done it subject to the restriction of using material developed so far in the text. But it is fun to try. I've had more correspondence about this problem than about any other point in the whole book." Incidentally, a solution to exercise **26, using only the tools that Prof. Herstein would have approved of, was featured in the December 2009 issue of the MONTHLY (see [**1**]).

REFERENCES

1. R. Beals, On orders of subgroups in abelian groups: An elementary solution of an exercise of Herstein. *Amer. Math. Monthly* **10** (2009). 923-926.
2. I. N. Herstein, *Topics in Algebra*. Second Edition. John Wiley & Sons, 1975.

Scott T. Chapman, *Editor*